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Dibaryons, strings, and branes in AdS orbifold models

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Abstract: A generalization of the Maldacena conjecture asserts that Type IIB string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ is equivalent to a certain supersymmetric $SU(N)^3$ gauge theory with bifundamental matter. To test this assertion, we analyze the wrapped branes on $\mathbf{S}^5/\mathbb{Z}_3$ and their interpretation in terms of gauge theory. The wrapped branes are interpreted in some cases as baryons or dibaryons of the gauge theory and in other cases as strings around which there is a global monodromy. In order to successfully match the brane analysis with field theory, we must uncover some aspects of S-duality which are novel even in the case of four-dimensional free field theory.

Keywords: Duality in Gauge Field Theories, Superstring Vacua, Brane Dynamics in Gauge Theories.

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1. Introduction

The AdS/CFT correspondence proposed by Maldacena [1] has made it possible to understand many aspects of the large N limit of conformal field theories in four dimensions via Type IIB compactifications on $AdS_5 \times X$. Here X is a compact Einstein manifold of positive curvature, and the conformal field theory is formulated on the boundary of AdS_5.

One aspect of the correspondence is that branes wrapped on nontrivial cycles in X can be compared to states in the conformal field theory that are nonperturbative from the point of view of the $1/N$ expansion. In [2], such an analysis was made for $X = \mathbf{S}^5$ and $\mathbb{RP}^5$; it was shown that the wrapped branes could be interpreted as soliton-like states – such as baryons, strings, and domain walls – in the large N gauge theory defined on the boundary. Some analogous results have been obtained in [3] for certain $\mathcal{N} = 1$ theories, and in [4] for a three-dimensional field theory.

Simple examples of X's with reduced supersymmetry can be constructed as orbifolds [5]. In this paper, we will consider in detail the example $X = \mathbf{S}^5/\mathbb{Z}_3$, with $\mathcal{N} = 1$ supersymmetry. One advantage of orbifolds is that it is comparatively easy to identify the boundary conformal field theory as a gauge theory. $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ with N units of flux on $\mathbf{S}^5/\mathbb{Z}_3$ is the near-horizon geometry of N parallel threebranes near a $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singularity. Putting N threebranes at this orbifold gives a system with [6] gauge group $(U(N))^3/U(1)$ and chiral multiplets transforming as

$$(\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{N}, \bar{\mathbf{N}}) \oplus (\bar{\mathbf{N}}, \mathbf{1}, \mathbf{N})\,. \tag{1.1}$$

There is also a cubic superpotential. A conformal field theory can hardly have $U(1)$ gauge fields coupled to chiral superfields, so we are led to suspect that in the AdS limit the $U(1)$ factors in $(U(N))^3/U(1)$ are decoupled. (For a dynamical explanation of this decoupling via anomalies, see [7, 8].) Thus, we suspect that the Type IIB superstring theory on $X = \mathbf{S}^5/\mathbb{Z}_3$ should be compared to an $SU(N)^3$ gauge theory with the same chiral multiplets as in 1.1. We henceforth call this theory simply the SCFT.

This paper will be devoted to a detailed comparison of the SCFT to Type IIB superstring theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$. A basic step in this comparison is to match up the symmetries of the two theories. This turns out to be surprisingly subtle; to correctly identify the global symmetry group on the string theory side depends on a surprising fact, which is that under certain conditions the operators measuring the number of D-strings and the number of fundamental strings do not commute. It is also necessary, of course, to take into account chiral anomalies on the SCFT side.

Once the symmetries are matched, it becomes much easier to compare the wrapped branes of the string theory with states that are nonperturbative (with respect to $1/N$) in the SCFT. We identify the wrapped branes with four kinds of objects in the SCFT, namely baryon vertices, particles, strings, and domain walls. To be more precise, the SCFT has states that one might call baryonic, or "dibaryonic" as they are built from fields charged under two different $SU(N)$'s. These states correspond to threebranes wrapping three-cycles in $\mathbf{S}^5/\mathbb{Z}_3$ and strings wrapping one-cycles. There are also membranes in AdS_5 formed by wrapping the threebranes on one-cycles and the fivebranes on three-cycles. These membranes can end on the boundary and so look like "strings" in the boundary theory. There are in all 27 kinds of such "gauge strings"; it turns out that for every element of the discrete internal symmetry group of the model, there is a string which produces that given symmetry element as monodromy. This understanding of the strings enables us also to fill in a gap in [2]. Finally, fivebranes wrapping the entire manifold X are interpreted as an external baryon vertex, and domain walls constructed from unwrapped threebranes have the property that the gauge group jumps (from $SU(N)^3$ to $SU(N \pm 1)^3$) in crossing such a wall, rather as in [2].

The paper is organized as follows. In section 2 we study the string theory on the manifold $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ and enumerate the possible brane wrapping states, guided by the study of non-trivial homologies of the manifold. We also talk about the SCFT and present an analysis of the global symmetries so as to have a complete set of quantum numbers classifying our states. Section 3 deals with the strings and the monodromies they produce. Details on the geometry of $\mathbf{S}^5/\mathbb{Z}_3$ are collected in the Appendix.

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2. The Model

2.1 The SCFT picture

Our first task will be to analyze the symmetries and operator content of the conformal field theory described in the introduction.

Consider Type IIB string theory on an orbifold $\mathbb{R}^4 \times \mathbb{C}^3/\Gamma$, with Γ being a discrete subgroup of the rotation group $SO(6)$ of $\mathbb{C}^3 = \mathbb{R}^6$. Upon placing N D3-branes at the origin of $\mathbb{C}^3/\Gamma$ and taking the near horizon limit as in [1], we obtain Type IIB string theory on $AdS_5 \times \mathbf{S^5}/\Gamma$. This construction was first analyzed in [5] , and subsequent generalizations were discussed in [9] . These models give simple examples in which the AdS/CFT correspondence can be extended to backgrounds with reduced supersymmetry. For example, if Γ is contained in an $SU(3)$ subgroup of $SO(6)$ but not in an $SU(2)$, then the model has $\mathcal{N} = 1$ supersymmetry in four dimensions.

The AdS/CFT correspondence relates the string theory on $AdS_5 \times \mathbf{S^5}/\Gamma$ to the gauge theory that gives a low-energy description of the system of N D3-branes at the orbifold singularity $\mathbb{C}^3/\Gamma$. That latter gauge theory can be identified by familiar orbifold methods [6], [10], [11].

In this paper we will focus on a simple special case: $\Gamma = \mathbb{Z}_3$ with the Γ action on the coordinates z_i of $\mathbb{C}^3$ being generated by

$$z_i \to \exp(2\pi i/3) z_i \,. \tag{2.1}$$

We will consider a system of N D3-branes on $\mathbb{C}^3/\Gamma$, which one can consider as coming from $3N$ such branes on the covering space $\mathbb{C}^3$. Going to the near horizon $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ geometry, there are N units of fiveform flux on $\mathbf{S}^5/\mathbb{Z}_3$:

$$\int_{\mathbf{S^5}/\mathbb{Z}_3} \frac{G_{(5)}}{2\pi} = N \,. \tag{2.2}$$

On the covering space $\mathbf{S}^5$ of $\mathbf{S}^5/\mathbb{Z}_3$, the number of flux quanta is $3N$. The subgroup of $SO(6)$ that commutes with Γ is $H = U(3)/\mathbb{Z}_3$, and this is realized as a global symmetry group of the model. The center of H acts as a $U(1)$ group of R-symmetries.

The system of N D3-branes at the orbifold singularity is governed by a $U(N)^3$ gauge theory. There are chiral superfields which should be classified as a representation of $U(N)^3 \times H$. Actually, it is useful to introduce the covering group $H' = U(3)$ of H. We have $H = H'/\Gamma$, where Γ is the group of cube roots of unity. The chiral multiplets transform in the $\mathbf{3}$ of H' tensored with the representation

$$(\mathbf{N}, \bar{\mathbf{N}}, 1) \oplus (1, \mathbf{N}, \bar{\mathbf{N}}) \oplus (\bar{\mathbf{N}}, 1, \mathbf{N}) \tag{2.3}$$

of $U(N)^3$. As explained in the introduction, we will assume that the $U(1)$ factors of the gauge group should be dropped before comparing to $AdS \times \mathbf{S}^5/\mathbb{Z}_3$, and that the

SCFT of interest is an $SU(N)^3$ gauge theory with the chiral superfields indicated in 2.3. It is interesting to note that if N is divisible by 3, then a central element of H' that is a cube root of unity is equivalent to a gauge transformation by an element of the center of $SU(N)^3$. Hence, in this case, the connected global symmetry group of the SCFT is the group H that acts geometrically on $\mathbb{C}^3/\mathbb{Z}_3$ and is hence manifest in Type IIB superstring theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$. However, if N is not divisible by 3, then no nontrivial element of H' is equivalent to a gauge transformation, and the connected global symmetry group of the SCFT really is the threefold cover H' of the geometrical symmetry group H. At the end of section 2.2, we will see how this comes about in string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$.

We denote the matter superfields of the $SU(N)^3$ theory as U_μ, V_μ, W_μ respectively, where $\mu \in \{1,2,3\}$ labels the $\mathbf{3}$ of H', while U, V, and W are associated with the three summands in 2.3. The most general cubic superpotential with $SU(N)^3 \times H'$ symmetry is

$$\mathcal{W} = \gamma \epsilon_{\mu\nu\rho} U^\mu V^\nu W^\rho \,, \tag{2.4}$$

with γ a constant. For the orbifold, this superpotential is actually present with nonzero γ.

So far we have considered only the connected part of the global symmetry group. In comparing to the string theory, it will be very important to also understand the discrete global symmetries.

One obvious symmetry is a cyclic permutation of the three $SU(N)$ factors in the gauge group, accompanied by $(U,V,W) \to (V,W,U)$. This gives a $\mathbb{Z}_3$ symmetry group, whose generator we will call A.

To look for more discrete symmetries, we consider $(U,V,W) \to (aU, bV, cW)$, where a, b, and c are complex numbers of modulus one. There is no essential loss in considering only choices of a, b, c under which the superpotential is invariant (since we have already identified R-symmetries), so we assume $abc = 1$. Absence of anomalies under $SU(N)^3$ instantons gives $(ab)^{3N} = (bc)^{3N} = (ca)^{3N} = 1$; using also $abc = 1$, we get $a^{3N} = b^{3N} = c^{3N} = 1$. Moreover, a transformation with $a^N = b^N = c^N = 1$ is equivalent to a gauge transformation by an element of the center of $SU(N)^3$. If we set $\zeta = \exp(2\pi i/3N)$, then (modulo gauge transformations), the interesting choices of a, b, and c are generated by $B : (a,b,c) = (\zeta, \zeta^{-1}, 1)$ and $C : (a,b,c) = (\zeta^{-2}, \zeta, \zeta)$. One has $B^3 = C^3 = 1$ (modulo gauge transformations) and of course B and C commute.

Now we find a very interesting detail. Modulo gauge transformations, A and C commute, but

$$AB = BAC \,. \tag{2.5}$$

Thus, A, B, and C generate a nonabelian group F with 27 elements. Actually, it is somewhat imprecise to call F a discrete symmetry group; this is so if N is divisible by 3, but otherwise C is equivalent modulo a gauge transformation to the element

T^N of H', where $T = e^{2\pi i/3}$. (In proving this, one must use the fact that if N is not divisible by 3, then $N^2 - 1$ is divisible by 3. It follows that $\exp(2\pi i((1/3N) - N/3))$ is an integral power of $\exp(2\pi i/N)$, and so is an element of the center of $SU(N)$.) We will not incorporate this in our terminology and will refer to F as a discrete symmetry group.

The group F admits the following action of $SL(2,\mathbb{Z})$ by outer automorphisms. An element

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{2.6}$$

of $SL(2,\mathbb{Z})$ acts by

$$A \to A^a B^b\,, \qquad B \to A^c B^d\,, \qquad C \to C\,. \tag{2.7}$$

(Of course, this transformation only depends on the reduction of M modulo 3). The model is expected to have an $SL(2,\mathbb{Z})$ S-duality symmetry, inherited from the $SL(2,\mathbb{Z})$ symmetry of Type IIB in ten dimensions. We propose that S-duality acts on the discrete symmetries in the way just indicated. This proposal will be incorporated in our proposal for matching the SCFT with Type IIB superstrings on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$.

The SCFT has a few other discrete symmetries that will be much less important in the present paper and which we note only briefly. If γ is real, then there is a parity symmetry P in which exchange of two factors in the gauge group is accompanied by orientation reversal of spacetime. There is also a charge conjugation symmetry C which exchanges two factors of the gauge group and acts in each factor of the gauge group as the outer automorphism that maps $\mathbf{N}$ to $\bar{\mathbf{N}}$. In the string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$, P corresponds to an orientation-reversing symmetry of $\mathbf{S}^5/\mathbb{Z}_3$ combined with one of AdS_5, and C to the world-sheet orientation reversal Ω.

Nonperturbative excitations in the $1/N$ expansion. Now let us discuss the spectrum of the SCFT. First, we consider states that are perturbative from the point of view of the $1/N$ expansion. These are states that can be built from a fixed number of elementary excitations, independent of N. If we let n_U be the number of U fields minus antifields (and including, of course, the fermionic partners of U), and define similarly n_V, n_W, then a simple exercise in $SU(N)^3$ group theory shows that all gauge-invariant excitations made from a fixed number of quanta (independent of N) have $n_U = n_V = n_W$. Hence, such excitations are invariant under B and C. On the other hand, A can perfectly well act nontrivially on perturbative excitations. Thus, the 27-element group F has a $\mathbb{Z}_3 \times \mathbb{Z}_3$ subgroup, generated by B and C, that acts trivially on states that are perturbative in the $1/N$ expansion.

What about nonperturbative states? One can build in this theory a gauge-invariant operator, nonperturbative with respect to the $1/N$ expansion, of the form $\epsilon^{i_1\ldots i_N}\epsilon_{j_1\ldots j_N}U^{j_i}_{i_1}\ldots U^{j_n}_{i_N}$. We schematically denote this state as U^N. One can build analogous states V^N and W^N. We will call these states baryons, or dibaryons. B and C act nontrivially on dibaryons.

In addition to these baryonic states (which are somewhat analogous to the Pfaffian states considered in [2] in the case of $SO(2n)$ gauge theory), one can consider baryon vertices connecting external charges. For example, N external charges in, say, the first $SU(N)$ factor in the gauge group can be combined to a gauge-invariant state using the antisymmetric tensor $\epsilon_{i_1 i_2 \ldots i_N}$. We will want to describe such a baryon vertex in terms of AdS_5.

At first sight, it may seem that there are three kinds of baryon vertex to consider — as one could have external charges in any of the three $SU(N)$ factors in the gauge group. However, modulo emission and absorption of the baryonic particles U^N, V^N, and W^N, the three types of baryon vertex are equivalent. For instance, since the U field transforms as $(\mathbf{N}, \bar{\mathbf{N}}, \mathbf{1})$ under $SU(N)^3$, a baryon vertex in the second $SU(N)$ plus a U^N state is equivalent to a baryon vertex in the first $SU(N)$.

2.2 String theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$

Type IIB on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ has obvious symmetries that come from geometrical symmetries of this manifold, namely the Anti-de Sitter symmetry group and the $H = U(3)/\mathbb{Z}_3$ symmetry group of $\mathbf{S}^5/\mathbb{Z}_3$. To identify additional symmetries of Type IIB superstring theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$, we must look at the possibilities of brane wrapping. The nontrivial integral homology groups of $\mathbf{S}^5/\mathbb{Z}_3$ are

$$\begin{aligned} H_0(\mathbf{S^5}/\mathbb{Z}_3, \mathbb{Z}) &= H_5(\mathbf{S^5}/\mathbb{Z}_3, \mathbb{Z}) = \mathbb{Z}\,, \\ H_1(\mathbf{S^5}/\mathbb{Z}_3, \mathbb{Z}) &= H_3(\mathbf{S^5}/\mathbb{Z}_3, \mathbb{Z}) = \mathbb{Z}_3\,. \end{aligned} \tag{2.8}$$

A generator of $H_1(\mathbf{S}^5/\mathbb{Z}_3)$ is a linearly embedded $\mathbf{S}^1/\mathbb{Z}_3$ subspace; a generator of $H_3(\mathbf{S}^5/\mathbb{Z}_3)$ is similarly a linearly embedded $\mathbf{S}^3/\mathbb{Z}_3$ subspace.[1]

The possibilities for brane wrapping are thus as follows:

1. We can make particles in AdS_5 by wrapping a p-brane on a p-cycle for $p = 1, 3, 5$. (In some cases, these objects actually turn out to be baryon vertices, connected to the boundary by strings, rather than localized particles.)

2. We can make membranes in AdS_5 by wrapping a p-brane on a $(p-2)$-cycle for $p = 3, 5$.

3. We can wrap fivebranes on one-cycles in $\mathbf{S}^5/\mathbb{Z}_5$ to make an object that completely fills AdS_5.

4. Finally, p-branes that are not wrapped at all on $\mathbf{S}^5/\mathbb{Z}_3$ look like p-branes on AdS_5.

[1] To be precise about this, let $\mathbf{S}^5$ be the subspace $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$ in $\mathbb{C}^3$, with $\mathbb{Z}_3$ acting by $z_i \to e^{2\pi i/3} z_i$. Then, up to a $U(3)$ transformation, $\mathbf{S}^1/\mathbb{Z}_3$ is defined by $z_2 = z_3 = 0$, and $\mathbf{S}^3/\mathbb{Z}_3$ is defined by $z_3 = 0$.

Wrapped branes of type 3 really correspond to having a different AdS_5 theory, giving something that should be compared not to the SCFT we have described, but to a different (possibly nonconformal) boundary theory. This possibly interesting direction will not be explored in the present paper. The unwrapped branes, type 4, are also easy to dispose of. The unwrapped onebranes are related to Wilson and 't Hooft loops in the boundary conformal field theory, as in [12], [13]. The unwrapped threebranes are domain walls, across which the $SU(N)^3$ theory jumps to an $SU(N \pm 1)^3$ theory, by the same reasoning as in [2]. We will concentrate in the present paper primarily on wrapped branes of types 1 and 2.

Concerning type 1, the fivebranes that are entirely wrapped on $\mathbf{S}^5/\mathbb{Z}_3$ can be interpreted precisely as in [2] in terms of baryon vertices connecting external quarks. To be specific, the totally wrapped $D5$-brane is a baryon vertex connected by elementary strings to N external electric charges; modulo emission and absorption of ordinary particles (localized AdS excitations), there is only one such vertex (rather than one for each factor in the gauge group) for reasons explained at the end of section 2.1.

The other objects of type 1 are a fundamental string or D-string wrapped on $\mathbf{S}^1/\mathbb{Z}_3$, and a threebrane wrapped on $\mathbf{S}^3/\mathbb{Z}_3$. The number of such wrapped objects (of any of the three kinds) is conserved modulo 3, since the relevant homology groups of $\mathbf{S}^5/\mathbb{Z}_3$ are both isomorphic to $\mathbb{Z}_3$. Let A' be the operator that counts wrapped fundamental strings (on a state with k such strings, the eigenvalue of A' is $\exp(2\pi ik/3)$), and similarly let B' and C' be the operators that count the numbers of wrapped D-strings and wrapped threebranes, respectively.

We would like to compare the symmetry generators called A', B', and C' here with the operators A, B, and C of the SCFT. We note that wrapped fundamental strings can be seen in string perturbation theory and so should correspond to perturbative objects in the $1/N$ expansion of the SCFT. By contrast, the other wrapped branes are nonperturbative objects in string perturbation theory and should be nonperturbative in the $1/N$ expansion of the boundary theory.

Hence, comparing to our analysis of the discrete symmetries of the SCFT in section 2.1, we identify A' with A. This identification can actually be justified directly by considering the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold. The orbifold has of course a quantum $\mathbb{Z}_3$ symmetry (which acts trivially on strings in the untwisted sector and nontrivially on twisted sectors). For N threebranes near the orbifold singularity, the quantum $\mathbb{Z}_3$ symmetry becomes [6] the group of cyclic permutations of the three $SU(N)$'s; the generator of this group is what we have called A. The twisted sector states, on which A acts nontrivially, become wrapped fundamental strings when we go to the near horizon $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ geometry, and this explains why $A = A'$.

Now, in section 2.1, we worked out the commutation relations of A, B, and C, and discovered an $SL(2, \mathbb{Z})$ group of outer automorphisms that intertwines A and B according to 2.7. In Type IIB superstring theory, there is an $SL(2, \mathbb{Z})$ S-duality group that intertwines in precisely the same way the operators A' and B' measuring the numbers of wrapped strings. We thus extend our identification of A with A' to identify B with B'.

Finally, by default, we are left to postulate that C should be identified with C'. (We will also give a fairly direct argument for this below.) Here, we must face the following puzzle. In string theory, it appears that the operators A', B', and C' measuring the numbers of wrapped branes of different kinds should all commute. They thus appear to generate the 27-element group $(\mathbb{Z}_3)^3$. However, in section 2.1, we learned that A, B, and C generate a *nonabelian* group with 27 elements. What is the origin of this discrepancy? In section 2.3 below, we will analyze this question and show that in fact, the operators A', B', and C' do not commute and obey instead

$$A'B' = B'A'C' \,. \tag{2.9}$$

Since (according to our hypothesis) $C' = 1$ in the absence of wrapped threebranes, and $C' = \exp(\pm 2\pi i/3)$ when a wrapped threebrane or anti-threebrane is present, the concrete meaning of this statement is that although A' and B' commute in the absence of a wrapped threebrane, they no longer commute in the presence of such a brane. Concretely, a wrapped threebrane supports a $U(1)$ gauge field, and suitable states of this gauge field carry F-string (fundamental string) and D-string number. The operators A' and B' become at low energies essentially the Wilson and 't Hooft loop operators of the $U(1)$ gauge theory. Thus, the assertion that A' and B' do not commute in the presence of the threebrane (but obey 2.9) will be justified by establishing a novel effect in free field theory, more specifically in $U(1)$ gauge theory in four dimensions.

The other main subject of the rest of this paper will be the wrapped branes of type 2, which give twobranes on AdS_5. These objects are of codimension two, so there can be a monodromy in going around such an object. Taking 0, 1, or 2 threebranes on $\mathbf{S}^1/\mathbb{Z}_3$, and 0, 1, or 2 Dirichlet or NS fivebranes on $\mathbf{S}^3/\mathbb{Z}_3$, with all these objects parallel to each other on AdS_5, we see that there are 27 possible membranes, counting the trivial one. 27 is the order of the global symmetry group F, and this suggests that each element of F is the monodromy around one of the membranes. If so, it is clear that C', which is $SL(2,\mathbb{Z})$-invariant, must be the monodromy around a membrane made by wrapping a threebrane, while A' and B' must be the monodromies around membranes constructed from wrapped fivebranes. Justifying these statements will be the goal of section 3.

Extension of the global symmetry group. To tie up some loose ends and further justify the identification of the threebrane wrapping number C' with C, we now examine the quantization of the wrapped threebrane. We want to see how the symmetry group $H = U(3)/\mathbb{Z}_3$ of $\mathbf{S}^5/\mathbb{Z}_5$ is extended to $H' = U(3)$, as predicted in section 2.1, when N is not divisible by 3.

A threebrane wrapped on a particular $\mathbf{S}^3/\mathbb{Z}_3 \subset \mathbf{S}^5/\mathbb{Z}_3$ is invariant under a subgroup $U(2)/\mathbb{Z}_3$ of $U(3)/\mathbb{Z}_3$. The space of such classical configurations is thus a copy of $(U(3)/\mathbb{Z}_3)/(U_2/\mathbb{Z}_3) = U(3)/U(2) = \mathbb{CP}^2$. The wrapped threebrane is thus equivalent at low energies (and large g^2N) to a particle moving on $\mathbb{CP}^2$. Because the threebrane is electrically charged with respect to N units of five-form flux, the wave function of this particle is a section of the line bundle $\mathcal{L}^N$, where $\mathcal{L} = \mathcal{O}(1)$ is the usual ample line bundle over $\mathbb{CP}^2$. The holomorphic sections of $\mathcal{L}^N$ – which give the lowest energy states of the wrapped threebrane – transform in the N^{th} symmetric tensor representation of $H' = U(3)$. This representation is faithful if N is not divisible by three, showing, as we saw in section 2.1 from the point of view of the SCFT, that for N not divisible by three, H is extended to H'.

We can be more specific about this. Supposing that N is not divisible by 3, let T be the element $\exp(2\pi i/3)$ of H'. Thus T measures the "triality" of an H' representation. T acts trivially on states that contain no wrapped threebranes. (It acts trivially on perturbative string states, since it acts trivially on the spacetime $AdS_5 \times \mathbf{S}_5$. Also, by quantizing the appropriate collective coordinates, it can be seen to act trivially on wrapped onebranes.) But on a state with a wrapped threebrane, T acts, given what we have seen in the last paragraph, as $\exp(2\pi i N/3)$. This is the same as the triality or T eigenvalue of the dibaryon state U^N, supporting the idea that the wrapped threebrane is a dibaryon. The relation between C' and T can be written $T = (C')^N$ or equivalently if N is not divisible by 3 (and hence N^2 is congruent to 1 modulo 3) $C' = T^N$. We found the same formula for C in section 2.1, supporting the relation $C' = C$.

2.3 Topologically nontrivial 't Hooft and Wilson lines

It remains to explain an important detail. As we have seen, for string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$ to agree with the SCFT, it must be that in the presence of a wrapped threebrane, the operators A' and B' measuring the number of wrapped fundamental strings or D-strings do not commute.

On the worldvolume of the wrapped threebrane – which for our purposes is a copy of $\mathbf{S}^3/\mathbb{Z}_3$ – there is a $U(1)$ gauge field a. Suitable configurations of this gauge field, roughly with nonzero eigenvalues of Wilson or 't Hooft loops, carry fundamental string or D-string charge. Thus, our question amounts to a question about free $U(1)$ gauge theory on $\mathbf{S}^3/\mathbb{Z}_3$.

In general, on a Type IIB threebrane of any topology, the induced fundamental string charge is measured by the first Chern class of the $U(1)$ line bundle. Complex line bundles on $\mathbf{S}^3/\mathbb{Z}_3$ are classified by their first Chern class which takes values in

$$H^2(\mathbf{S}^3/\mathbb{Z}_3, \mathbb{Z}) = \mathbb{Z}_3\,. \tag{2.10}$$

Thus, as expected, the fundamental string charge on $\mathbf{S}^3/\mathbb{Z}_3$ is $\mathbb{Z}_3$-valued.

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A line bundle whose first Chern class is torsion admits a flat connection. The fundamental group of $\mathbf{S}^3/\mathbb{Z}_3$ is $\mathbb{Z}_3$, and a flat connection is specified by up to isomorphism by giving its monodromy around a circle α generating this $\mathbb{Z}_3$. The monodromy is of the form $\exp(2\pi ik/3)$, where $k = 0, 1$, or 2 is the fundamental string number. Thus, for each line bundle, there is a minimum energy state, associated with the flat connection on that line bundle. For the flat connection on the k^{th} line bundle, the value of the Wilson line $W = \exp i \int_\alpha a$ is $\exp(2\pi ik/3)$, and this is the expected eigenvalue in that sector of the operator A that counts fundamental wrapped strings. In that sense, the fundamental string operator is related to the Wilson line.

Dually, we expect to measure the number of wrapped D-strings by an 't Hooft loop operator on the circle α. Here we will meet a very interesting subtlety which will lead to the expected formula $AB = BAC$. The subtlety has apparently been unnoticed before because 't Hooft loops associated with homologically non-trivial cycles such as α have not been much studied.

The standard definition of the 't Hooft loop is as follows. We state the recipe for a general three-manifold M and a circle $\alpha \subset M$. Let $S = e^{i\phi}$ be a $U(1)$-valued function on $M - \alpha$ (the complement of α in M) that has "winding number one" around α. This means that S changes in phase by 2π in going around a small circle β that has linking number one with α. The 't Hooft loop operator is then defined as a gauge transformation by S; under this transformation, one has $a \to a - id\ln S = a - d\phi$.

The problem with this definition is that S is described near α, but there is no recipe for what S should look like far away from α. As we will see, when α is homologically nontrivial, a $U(1)$-valued function S with the claimed properties does not exist. This problem does not arise in most previous studies of 't Hooft loops because homologically trivial α's have most often been considered. (If α is the boundary of an oriented two-manifold $D \subset M$, one can give a recipe for defining S globally with the desired properties, such that $S = 1$ except very near D.) On the other hand, in most studies of 't Hooft loops, fractional magnetic charge is considered (for the present case of $U(1)$ gauge theory, this means that S is multivalued in going around α, with the change in phase being a fractional multiple of 2π). One then gets interesting properties such as the celebrated commutation relations of 't Hooft and Wilson loops. In our present problem, the electric and magnetic charges are integral, but the cycles are nontrivial. This will lead to somewhat analogous results.

Before explaining why S does not exist if one expects it to be $U(1)$-valued, let us first explain in what sense S does exist. Given any codimension-two cycle α in a manifold M, one can define the Poincaré dual cohomology class $[\alpha] \in H^2(M, \mathbb{Z})$, and a complex line bundle $\mathcal{L}$, unique up to isomorphism, with $c_1(\mathcal{L}) = [\alpha]$. Moreover, $\mathcal{L}$ has a smooth section s with a simple zero along α, of winding number 1 around α. Now, on the complement of α, define S by $S = s/|s|$. S has the desired properties $|S| = 1$ and winding number one around α, but S is a section of $\mathcal{L}$ rather than a $U(1)$-valued function.

At this point, we can readily show the converse: if $[\alpha] \neq 0$, then S does not exist as an ordinary function. Let S' be a hypothetical $U(1)$-valued function with winding number one around α. Then S/S' has no winding number around α, and hence extends over α as a smooth and everywhere nonzero section of $\mathcal{L}$. Such a function is a trivialization of $\mathcal{L}$. So S' can only exist if $\mathcal{L}$ is trivial, or in other words if $[\alpha] = 0$.

If $[\alpha] \neq 0$ and we define an 't Hooft loop using a "gauge transformation" by S, what will we get? A charged field Ψ will be transformed by this "gauge transformation" to $S\Psi$. If Ψ is a section of a line bundle $\mathcal{M}$, then $S\Psi$ is a section of $\mathcal{L}\otimes\mathcal{M}$. The operation $\mathcal{M} \to \mathcal{L} \otimes \mathcal{M}$ shifts the first Chern class of $\mathcal{M}$ by $c_1(\mathcal{L})$. In our problem, the first Chern class of $\mathcal{M}$ is understood as fundamental string winding number, so the "gauge transformation" by S shifts that winding number.

For this reason, the 't Hooft loop operator on $\mathbf{S}^3/\mathbb{Z}_3$ does not commute with the elementary string winding number. If as above we measure the elementary string winding number by an operator A' that takes the value $\exp(2\pi i k/3)$ when the first Chern class is k, and define the D-string winding number by a 't Hooft loop operator B' that increases k by 1, then we get the expected commutation relation $A'B' = B'A'\exp(2\pi i/3)$ for states with a single wrapped threebrane.

The issue we have investigated is actually relevant to a previous study [14] of threebrane wrapping on $\mathbf{S}^3/\mathbb{Z}_n$. In that work, it was important that n states can be made by letting the threebrane absorb k fundamental strings for $k = 0, 1, \ldots, n-1$, and that states with absorbed D-strings should not be counted separately. The relation $A'B' = B'A'\exp(2\pi i/n)$ that follows from the above analysis makes clear why this is so. If $|k\rangle$ is a state with k absorbed fundamental strings, then a state with k' absorbed D-strings is $\sum_k \exp(2\pi i k k'/n)|k\rangle$. One cannot specify both the number of absorbed fundamental strings and the number of absorbed D-strings, since the relevant operators do not commute.

The group generated by two operators A, B with $AB = BA\exp(2\pi i/3)$ has, up to isomorphism, only one irreducible representation, which is of dimension three. From the point of view of the SCFT, the dibaryons U^N, V^N, and W^N are three states that transform in this representation. From the point of view of the string theory on $AdS_5 \times \mathbf{S}^5/\mathbb{Z}_3$, the wrapped threebrane, with its three possible flat $U(1)$ connections, has three ground states that transform in this representation.

3. Strings and monodromies

3.1 Preliminaries

So far our main focus has been point-like states in the boundary SCFT associated with branes that wrap various cycles in the internal space $X = \mathbf{S}^5/\mathbb{Z}_3$. We have also briefly discussed a few other types of brane configurations. What remains is to discuss membrane-like objects in AdS_5 that look like "strings" on the boundary. As discussed in section 2.2, these can arise from *(a)* NS5-branes wrapping a 3-cycle, *(b)* D5-brane wrapping a 3-cycle, or *(c)* D3-branes wrapping a 1-cycle in X. Since the membranes are of codimension two in AdS_5, it is possible to have a monodromy when a particle is taken around a membrane. As we noted in section 2.2, each type of membrane is classified by a $\mathbb{Z}_3$ charge, and a total of 27 membranes (counting the trivial one) can be constructed. This suggests that the monodromies might comprise the nonabelian group F of 27 elements.

The particles that we will use as test objects to compute monodromies are the wrapped onebranes and threebranes that have already been studied in section 2. The monodromies will arise from two different effects. One is simply that the wrapped branes that give membranes and the ones that give test particles have electric and magnetic couplings to p-form gauge fields; these couplings lead to numerical monodromies. The other effect, seen in the threebrane-fivebrane system, is somewhat more exotic and involves a certain brane creation process [15]. What happens here is that when a wrapped threebrane goes around a membrane made by wrapping a fivebrane, it returns to itself with creation of a string.

A useful technical aid in the computation is the following. The discussion of monodromies will be purely topological, so AdS_5 can be replaced by $\mathbb{R}^5$ (with which it coincides topologically). The membrane worldvolume M can be taken to be a copy of $\mathbb{R}^3 \subset \mathbb{R}^5$. As for the particle worldline, in computing monodromies one takes it to be a circle $C = \mathbf{S}^1$ that winds once around the $\mathbb{R}^3$. The essential property of the situation is thus that M and C are linked. To exhibit this linking neatly, it is convenient to compactify $\mathbb{R}^5$ to $Y = \mathbf{S}^5$, in which case M can be compactified to $\mathbf{S}^3$. Thus M and C are respectively copies of $\mathbf{S}^3$ and $\mathbf{S}^1$ in Y, and topologically M and C are linked. The linking means that a manifold $B \subset Y$ with boundary M has intersection number 1 with C, and conversely a manifold $B' \subset Y$ with boundary C has intersection number 1 with M.

As we noted in section 2.2, if the monodromies around the membranes are to generate the group F, then the central element C of F must be generated by a membrane of type *(c)*. Since C counts dibaryons and acts trivially on everything else, we expect to find that the monodromy in going around a membrane of type *(c)* is a factor of $e^{2\pi i/3}$ if the test particle is a threebrane wrapped on $\mathbf{S}^3/\mathbb{Z}_3$, and is otherwise trivial. On the other hand, membranes of type *(a)* and *(b)* must give monodromies A and B. A monodromy A, for example, assigns a phase $e^{2\pi i/3}$ if the test particle is a wrapped fundamental string, is trivial if the test particle is a wrapped D-string, and (in view of the action of A and B on dibaryons) is more interesting and will be described later if the test particle is a wrapped threebrane. A monodromy B is similar with F-strings and D-strings exchanged.

3.2 Aharonov-Bohm effect for branes

We consider first the monodromies that arise just from electric and magnetic couplings of the test particle and the membrane to the same p-form gauge field. (In view of the discussion in the last paragraph, this means everything except the parallel transport of a dibaryon state around a membrane made by wrapping a fivebrane.) How do electric and magnetic couplings give monodromies? The most elementary example, which we will generalize, is the standard Aharonov-Bohm effect in QED. There we have a topologically non-trivial field configuration, where on taking a particle with unit charge in a closed loop C around a magnetic source, we pick up a phase given by $\exp(i \int_C A \cdot dx)$. Basically the phase is measured by the line integral of the vector potential over the world-line of the particle. There is a higher dimensional example for branes; a p-brane with worldvolume V coupled to a $(p+1)$-form gauge field A has a worldvolume interaction $\int_V A$. This will create a monodromy if the p-brane is parallel transported around a suitable magnetic source of A. When we parallel transport a probe (made by wrapping a brane on a cycle in $\mathbf{S}^5/\mathbb{Z}_3$) around an AdS_5 membrane (made by wrapping another brane on another cycle in $\mathbf{S}^5/\mathbb{Z}_3$), we can get a nontrivial monodromy by this mechanism if the two branes couple electrically and magnetically to the same field. This will occur in the following cases:

(a′) A membrane of type *(a)*, made from a wrapped NS5-brane, and a probe made from a dual fundamental string wrapped on $\mathbf{S}^1/\mathbb{Z}_3$.

(b′) A membrane of type *(b)*, made from a wrapped D5-brane, and a probe made from a dual D-string wrapped on $\mathbf{S}^1/\mathbb{Z}_3$.

(c′) A membrane of type *(c)*, made from a wrapped D3-brane, and a probe made from a dual D3-brane wrapped on $\mathbf{S}^3/\mathbb{Z}_3$.

The monodromies that we will get from these cases are all the required monodromies summarized at the end of section 3.1 except for the more complicated mondromy involving a probe D3-brane and a membrane of type *(a)* or *(b)*. We postpone considering this last case.

For analyzing the Aharonov-Bohm c-number monodromies, we consider for definiteness case *(a′)*. It will be evident that the other cases are similar.

As explained in section 3.1, we can replace AdS_5 by $\mathbf{S}^5$ for the present purposes. We thus think of the spacetime as $\mathbf{S}^5 \times \mathbf{S}^5/\mathbb{Z}_3$. We consider a fundamental string whose worldvolume is $V_s = C \times \mathbf{S}^1/\mathbb{Z}_3$, where $C \subset \mathbf{S}^5$ and $\mathbf{S}^1/\mathbb{Z}_3$ is as usual a generator of $H_1(\mathbf{S}^5/\mathbb{Z}_3)$. Likewise, we consider an NS5-brane with worldvolume $V_m = M \times \mathbf{S}^3/\mathbb{Z}_3$, with $M \subset \mathbf{S}^5$ and $\mathbf{S}^3/\mathbb{Z}_3$ a generator of $H_3(\mathbf{S}^5/\mathbb{Z}_3)$. As explained in section 3.1, C and M are a circle and a three-sphere which are "linked" in $\mathbf{S}^5$.

The fivebrane is a magnetic source of the Neveu-Schwarz two-form field B. The factor in the path integral that will give the monodromy T is, roughly speaking,

$$T = \exp\left(i \int_{V_s} B_w\right), \tag{3.1}$$

where B_w is the B-field created by the fivebrane. The reason that this is only roughly the right formula is that the B-field created by the fivebrane is topologically nontrivial and so cannot be represented globally by a two-form B_w. A safe way to proceed is to let Z_s be a three-manifold with boundary V_s and rewrite 3.1 as

$$T = \exp\left(i\int_{Z_s} F_w\right), \tag{3.2}$$

where $F_w = dB_w$ is the gauge-invariant threeform field created by the fivebrane. This is a better formula because F_w is gauge-invariant and is globally defined.

F_w is determined by the following conditions. First,

$$dF_w = 2\pi\delta(Z_m), \tag{3.3}$$

where $\delta(Z_m)$ is understood as a four-form Poincaré dual to the six-manifold Z_m; in what follows, analogous delta functions will be understood similarly. Second, F_w should obey the three-form analog of Maxwell's equations.

If $\mathbf{S}^3/\mathbb{Z}_3$ were a boundary in $\mathbf{S}^5/\mathbb{Z}_3$, say the boundary of a four-manifold N, we could obey 3.3 with $F_w = 2\pi\delta(N)$. This is not actually so. However, three times $\mathbf{S}^3/\mathbb{Z}_3$ vanishes in $H_3(\mathbf{S}^5/\mathbb{Z}_3, \mathbb{Z})$ (since that group is $\mathbb{Z}_3$), so we can find a four-manifold $N \subset \mathbf{S}^5/\mathbb{Z}_3$ whose boundary is three copies of $\mathbf{S}^3/\mathbb{Z}_3$. With such an N, we can obey 3.3 with $F_w = (2\pi/3)\delta(N)$. The formula for the monodromy is now

$$T = \exp\left((2\pi i/3)\int_{Z_s} \delta(N)\right). \tag{3.4}$$

The integral $\int_{Z_s} \delta(N)$ counts the intersection number of Z_s and N. That intersection number is 1 modulo 3, since B has intersection number 1 with M (as C and M are linked in $\mathbf{S}^5$), and $\mathbf{S}^1/\mathbb{Z}_3$ has intersection number 1 modulo 3 with N (as $\mathbf{S}^1/\mathbb{Z}_3$ and $\mathbf{S}^3/\mathbb{Z}_3$ are similarly linked in $\mathbf{S}^5/\mathbb{Z}_3$). Hence the monodromy is $T = \exp(2\pi i/3)$.

This is the expected monodromy from the discussion at the end of section 3.1. It reflects the following facts: the monodromy for a membrane of type *(a)* is A; the eigenvalue of A for a wrapped fundamental string is $\exp(2\pi i/3)$. If we use for the test particle a wrapped D-string, a calculation similar to the above gives a trivial monodromy around the membrane of type *(a)* (since the D-string does not couple to the B-field created by the NS5-brane). For a test particle consisting of a wrapped threebrane, additional considerations that we come to shortly are relevant.

The other purely numerical monodromies – a membrane of type *(b)* and test particle a wrapped string, or a membrane of type *(c)* and any test particle – can be treated similarly. In each case, there is a nontrivial monodromy precisely if the test particle is electric-magnetic dual to the membrane.

The remaining case. It remains only to analyze the more elaborate monodromy that arises when a test particle made from a wrapped threebrane is transported around a membrane made from a wrapped fivebrane. For definiteness, we will consider the case of a membrane of type *(b)*, made from a wrapped D5-brane. We expect the monodromy to equal B.

We assume that the threebrane that we use as a test particle is prepared in an eigenstate of A, the operator that equals $\exp(2\pi ik/3)$ for a state with k wrapped fundamental strings. Since the wrapped threebrane has $C = \exp(2\pi i/3)$, the relation $AB = BAC$ means that (if the monodromy is equal to B) the wrapped threebrane, when transported around the membrane, returns with an extra wrapped fundamental string.

We again consider spacetime to be $\mathbf{S}^5 \times \mathbf{S}^5/\mathbb{Z}_3$. We take the threebrane worldvolume to be $Z_3 = C \times \mathbf{S}^3/\mathbb{Z}_3$ and the fivebrane worldvolume to be $Z_5 = M \times \mathbf{S}^3/\mathbb{Z}_3$. C and M are as before a circle and a three-sphere in $\mathbf{S}^5$. We take the two $\mathbf{S}^3/\mathbb{Z}_3$'s (the second factors in Z_3 and Z_5) to be distinct and generic. They then intersect on a circle $Y' \subset \mathbf{S}^5/\mathbb{Z}_3$ that is a copy of $\mathbf{S}^1/\mathbb{Z}_3$.

We compare two cases. In case (1), C and M are unlinked in $\mathbf{S}^5$, and in case (2), which is the real case of interest, they have linking number 1. We assume that in case (1), no fundamental strings are present. As one deforms from case (1) to case (2), C passes through M, meeting it (at some stage) at some point $P \in \mathbf{S}^5$, whereupon Z_3 and Z_5 meet on the circle $P \times Y'$. In passing through this intersection to get to case (2), a fundamental string is created, connecting the threebrane to the fivebrane, according to a process described in [15]. In the final state, the worldvolume of this string is (up to homotopy) $Q \times Y'$, with Q a path in $\mathbf{S}^5$ from C to M. This means that, in the monodromy described by case (2), at some moment in parallel transport about the membrane, the wrapped threebrane probe has absorbed an elementary string wrapped on $Y = \mathbf{S}^1/\mathbb{Z}_3$. This is the expected monodromy.

It remains to justify the assumption that in case (1), there are no net fundamental strings connecting the threebrane to the fivebrane. This is so for the following reason. A wrapped fundamental string ending on the threebrane carries electric charge (with respect to the $U(1)$ gauge field on the threebrane worldvolume Z_3); the total electric charge absorbed on Z_3 must vanish as Z_3 is compact. In case (2), the B-field of the fivebrane makes an extra contribution to the absorbed electric charge, but in case (1) it does not. Indeed in case (1), by adding an exact form to the B-field of the fivebrane, one can make this B-field vanish identically near the threebrane.

3.3 Tying up a loose end

Finally, we would like to tie up a loose end in [2].

In that paper, wrapped branes in $AdS_5 \times \mathbf{S}^5$ and $AdS_5 \times \mathbb{RP}^5$ were considered. Most of them were successfully compared to boundary conformal field theory. But there was one case for which no interpretation was offered – a threebrane wrapped on a generator of $H_1(\mathbb{RP}^5, \mathbb{Z}) = \mathbb{Z}_2$ to give a membrane in AdS_5, which we will call $\mathcal{M}$. In keeping with what has been seen above, one would guess that the proper interpretation of $\mathcal{M}$ is that there is a monodromy under transport around $\mathcal{M}$ consisting of some $\mathbb{Z}_2$ symmetry τ of the theory.

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The $AdS_5 \times \mathbb{RP}^5$ model depends on $\mathbb{Z}_2$-valued discrete theta angles θ_{NS} and θ_{RR} which were described in [2]. The gauge group is $SO(2k)$ (for some k) if $\theta_{NS} = \theta_{RR} = 0$ and otherwise is of the form $SO(2k+1)$ or $Sp(N)$. As we will explain presently, the membrane $\mathcal{M}$ is stable if and only if $\theta_{NS} = \theta_{RR} = 0$. So τ should be a discrete symmetry that exists when the gauge group is $SO(2k)$ but not when it is $SO(2k+1)$ or $Sp(k)$. There is an obvious candidate for such a discrete symmetry, namely the outer automorphism of $SO(2k)$ generated by a reflection in one of the coordinates. $SO(2k+1)$ and $Sp(N)$ have no such outer automorphism.

The outer automorphism of $SO(2k)$ actually played an important role in [2]. The "Pfaffian particle" (constructed by wrapping a threebrane on a generator of $H_3(\mathbb{RP}^5, \mathbb{Z}) = \mathbb{Z}_2$) is odd under this outer automorphism, and so should have a monodromy -1 under parallel transport around $\mathcal{M}$. This can be seen by an Aharonov-Bohm effect analogous to what was explained above.

It remains to explain why $\mathcal{M}$ is unstable unless $\theta_{NS} = \theta_{RR} = 0$. A membrane is unstable if it can end on a string, for then it decays by nucleation of string loops. In [2], strings in AdS_5 made by wrapping fivebranes on fourcycles in $\mathbb{RP}^5$ were considered. It was shown that for $\theta_{NS} = \theta_{RR} = 0$, one can make a string by wrapping either an NS5-brane or a D5-brane on an $\mathbb{RP}^4 \subset \mathbb{RP}^5$. However, if $(\theta_{NS}, \theta_{RR}) \neq (0,0)$, then one or the other kind of string is absent. This arises as follows. Consider, for example, a string made by wrapping an D5-brane. Let $[H]$ be the cohomology class of the NS B-field. On the worldvolume V_5 of a D5-brane, one requires (in the absence of threebranes)

$$[H]|_{V_5} = 0\,. \tag{3.5}$$

For a D5-brane wrapped on $\mathbb{RP}^4$, this condition is obeyed if and only if $\theta_{NS} = 0$. If threebranes, ending on V_5 in a three-manifold D, are included, the condition 3.5 becomes

$$[H]|_{V_5} + [D] = 0\,, \tag{3.6}$$

with $[D]$ the Poincaré dual to D. Applied to a D5-brane wrapped on $\mathbb{RP}^4$, this condition states that $[D]$ must be nonzero, and more specifically that the string made by wrapping the D5-brane must be the boundary of a membrane made from a wrapped threebrane; this is the membrane that we have called $\mathcal{M}$. Reading this statement in reverse, $\mathcal{M}$ can end on a string made from a wrapped D5-brane if $\theta_{NS} \neq 0$. By similar reasoning, $\mathcal{M}$ can end on a string made from a wrapped NS5-brane if $\theta_{RR} \neq 0$. In either case, $\mathcal{M}$ is unstable.

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A. Topology of the Lens spaces

Here we will, for completeness, compute the cohomology and homology groups of $X = \mathbf{S}^5/\mathbb{Z}_3$. Since this space is path connected and orientable, $H_0(X,\mathbb{Z}) = H_5(X,\mathbb{Z}) = \mathbb{Z}$. Because its universal cover is simply $\phi\colon \mathbf{S}^5 \longrightarrow X$, the fundamental group is $\pi_1(X) = \mathbb{Z}_3$ and therefore $H_1(X,\mathbb{Z}) = \mathbb{Z}_3$.

To learn more, we study X by viewing it as a Hopf-like fibration $\psi\colon X \longrightarrow \mathbb{CP}^2$. Indeed, the 5-sphere

$$\mid z_1 \mid^2 + \mid z_2 \mid^2 + \mid z_3 \mid^2 = 1 \tag{A.1}$$

admits a $U(1)$ symmetry $z_i \to e^{i\alpha} z_i$. This commutes with the action of $\mathbb{Z}_3$ on $\mathbf{S}^5$ (which is obtained by restricting $e^{i\alpha}$ to be a cube root of 1), and so descends to a $U(1)$ action on X. The quotient $X/U(1)$ is $\mathbb{CP}^2$. The cohomology of X can be obtained by a spectral sequence using this fibration; the computation is described in [16], p. 244. The result is that, apart from $H^0(X,\mathbb{Z}) = H^5(X,\mathbb{Z}) = \mathbb{Z}$, the nonzero integral cohomology groups are $H^2(X,\mathbb{Z}) = H^4(X,\mathbb{Z}) = \mathbb{Z}_3$. It then follows from the Universal Coefficient Theorem (Corollary 15.14.1 in [16]) that the nonzero homology groups of X, apart from H_0 and H_5, are $H_1(X,\mathbb{Z}_3) = H_3(X,\mathbb{Z}_3) = \mathbb{Z}_3$.

In turn, a three-cycle $Y = \mathbf{S}^3/\mathbb{Z}_3$ in X is a lens space itself. Its cohomology can be computed similarly (using the Hopf fibration over $\mathbb{CP}^1$), and in particular $H^2(Y,\mathbb{Z})$, which classifies complex line bundles over Y, is isomorphic to $\mathbb{Z}_3$.

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